IYSE 6420 Fall 2020 Homework2

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1. 2-D Density Tasks

**(a) marginal distribution fX(x) is exponential E(λ).**

Exponential:

When

So is exponential

**(b) marginal distribution fY (y) is Gamma Ga(2, λ).**

Gamma:

When ,

Let

So is Gamma

**(c) conditional distribution is shifted exponential,**

**(d) conditional distribution is uniform**

2. Weibull Lifetimes

Gamma:

**(a) For the prior suggested by the expert, find the posterior distribution of θ.**

Prior

Likelihood

Posterior

Let

**(b) What are the posterior mean and variance? No need to integrate if you recognize to which family of distributions the posterior belongs.**

The posterior is Gamma distribution,

Mean

Variance

3. Silver-Coated Nylon Fiber

**(a) Suppose λ = 1/5, find the probabilities that**

Probability density function

Cumulative distribution function

**(i) a run continues for at least 5 hours.**

**(ii) a run lasts less than 10 hours.**

**(iii) a run continues for at least 10 hours, given that it has lasted 5 hours.**

Since

**(b) Now suppose that the rate parameter λ is unknown, but there are three measurements of interblockage times, T1 = 2, T2 = 4, and T3 = 8.**

**(i) How would classical statistician estimate λ ?**

Classical statistician takes no prior knowledge into consideration

**(ii)What is the Bayes estimator of λ if the prior is**

Prior

Likelihood

Posterior

Gamma:

Let , we can see the posterior distribution is Gamma.

Bayes estimator